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Estimativa da densidade de potência eólica em cidades do nordeste do Brasil

Wind power density estimation for cities in the brazilian northeast

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Resumo: Melhorar a exatidão da estimativa de produção de energia eólica é preponderante para o planejamento estratégico no setor elétrico de uma nação. Nesse contexto, esta pesquisa teve como objetivo estimar os parâmetros do modelo estatístico de Weibull e a densidade de potência eólica usando dados coletados de três cidades no nordeste do Brasil. Além disso, outro objetivo foi analisar o melhor ajuste entre a distribuição dos dados observados e o modelo de Weibull. Para atingir esses objetivos, quatro metodologias distintas, a saber, Método de Regressão de Mínimos Quadrados (MRMQ), Método de Momentos (MM), Método de Fator de Padrão de Energia (MFPE) e Método de Máxima Verossimilhança (MMV), foram empregadas para estimar os parâmetros de forma e escala do modelo de Weibull. A fim de analisar o melhor ajuste entre os dados observados do vento e o modelo estatístico de Weibull, foi aplicado o teste estatístico: Erro Médio Quadrático (EMQ). Por sua vez, os valores médios dos parâmetros estimados obtidos através das quatro metodologias foram utilizados para calcular a densidade de potência eólica em cada cidade investigada. Os resultados deste estudo mostram que os ventos que sopram no nordeste do Brasil são de excelente qualidade favorecendo, a geração eólica. Além disso, todos os métodos examinados (ou seja, MRMQ, MMV, MM e MFPE) demonstraram desempenho satisfatório na estimativa dos parâmetros da distribuição de Weibull.
Palavras-chave: Energia, Parâmetros, Métodos, Weibull e Ajuste.

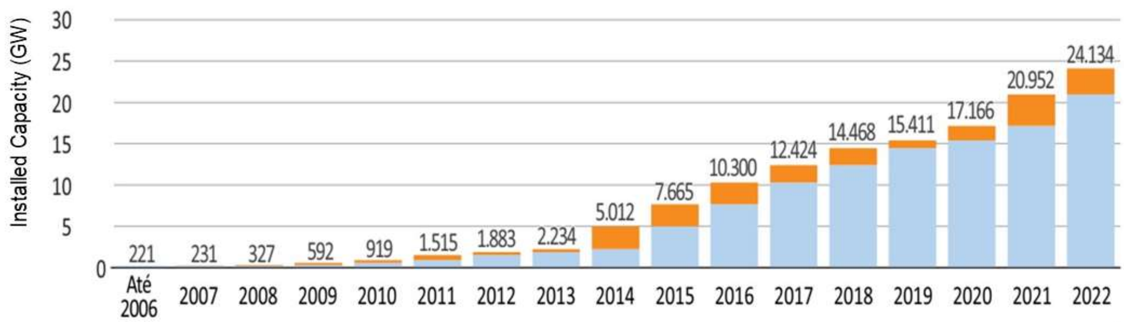
Abstract: *Improving the accuracy of wind energy production estimation is preponderant for strategic planning within a nation's electrical sector. In this context, this research aimed to estimate both the parameters of the Weibull statistical model and wind power density by using wind data collected from three cities in northeastern Brazil. Furthermore, another objective was the analysis of the best fit between the observed data distribution and the Weibull model. To achieve these goals, four distinct methodologies, namely, the Least-Squares Regression Method (LSRM), Moments Method (MOM), Energy Pattern Factor Method (EPFM), and Maximum Likelihood Method (MLM), were employed to estimate the shape and scale parameters of the Weibull model. In order to analyze the best fit between the observed wind data and the Weibull statistical model, it is imperative to apply at least one statistical test. Specifically, the selected statistical test was the Root Mean Square Error (RMSE). In turn, the mean values of the estimated parameters obtained through the four methods were employed to calculate the wind power densities in each investigated city.* The results of this study have shown that the winds blowing in the northeast of Brazil are of excellent quality, favoring wind power generation. *Furthermore, all the examined methods (i.e., LSRM, MLM, MOM, and EPFM) demonstrated satisfactory performance in estimating the parameters of the Weibull distribution.*
Keywords: Energy, Parameters, Methods, Weibull and Fit.

1 Introduction

Wind velocity is a stochastic variable used to produce wind energy by harnessing the kinetic energy of air in motion.. The airflow is converted into electrical energy using wind turbines or wind energy conversion systems. In turn, the production of wind power has grown in Brazil and worldwide to satisfy the high demand of the electricity sector for more electricity. As the demand for electricity has increased a lot, the installed capacity of wind farms has grown strongly over the years (Wwea, 2023). In particular, electricity demand in Brazil is forecasted to increase at an average rate of 2.5% per year in 2024-2026, compared to 2.2% over the 2018-2023 period. This growth is supported by continued brisk economic activity and higher residential consumption (Iea, 2024). According to the International Renewable Energy Agency, there was an increase in the global installed capacity for the production of wind energy of about 22.9% in the period between 2020 and 2022 (Irena, 2023).

According to the National Electric Energy Agency, until the first half of 2019, wind energy represented about 8.6% of the Brazilian electricity matrix and continues to grow (Aneel, 2019). More recently, in 2022, according to the report from the Brazilian Association of Wind Energy and New Technologies, Brazil has distinguished itself on the global stage for its electrical matrix strongly based on renewable sources, constituting approximately 90% of the total. It is noteworthy that, in Brazil, hydropower represents around 54.1% of the electrical matrix, while wind energy accounts for about 13.4%. The year 2022 ended with 904 wind farms and 25.63 GW of installed wind power capacity, representing a growth of 18.85% compared to December 2021 when the installed capacity was 21.56 GW (Brazilian association of wind power and new technologies, 2022).

Although wind energy is not the largest renewable source in the Brazilian electricity matrix, it continues to grow strongly due to the implementation of new wind farms. The new implemented wind farms have a direct impact on the growth of installed capacity for wind power generation. In turn, the evolution of installed wind power generation capacity is presented through Figure 1, where it is observed that the installed capacity reached 24 GW in 2022 (Etene, 2023). More recently, in 2023, installed capacity in Brazil reached slightly above 30 GW (Gwec, 2024).



	Até 2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
New Installed Capacity	-	10	96	265	327	596	368	351	2.778	2.653	2.635	2.124	2.045	943	1.755	3.786	3.182
Installed Capacity (previous year)	221	221	231	327	592	919	1.515	1.883	2.234	5.012	7.665	10.300	12.424	14.468	15.411	17.166	20.952
Total Installed Capacity	221	231	327	592	919	1.515	1.883	2.234	5.012	7.665	10.300	12.424	14.468	15.411	17.166	20.952	24.134

Figure 1. Evolution of installed Wind Power Generation Capacity in Brazil (GW). Source: Etene (2023). Adapted by the author.

All of the information above has motivated the development of this research, as it has highlighted the significant relevance of studies related to the estimation of local wind power density. In this context, the estimation of parameters from the Weibull probability distribution has been of fundamental importance prior to estimating the local wind power density (WPD). In particular, the Weibull probability distribution is one of the widely used distributions in technical practice. It is often used in weather forecasting, reliability theory, and lifetime analysis. This distribution was first introduced by the Swedish scientist Waloddi Weibull (1887-1979), who used it in reliability theory (Weibull, 1951; Pobocíková and Sedliacková, 2014).

Moreover, they can be estimated by various methods, all of which should aim to maximize the accuracy of the wind power density (WPD) estimate.

Shu and Jesson (2021), relying on wind speed and direction data from 38 meteorological stations located in the United Kingdom for the period 1981 to 2018, present both the assessment of wind characteristics through the Weibull probability distribution and the estimation of wind power density. In turn, the authors employed four methods to estimate the Weibull parameters (i.e., Empirical Method of

Justus, Lysen Method, Maximum Likelihood Method, and Energy Pattern Factor Method). In another study, the authors utilized wind speed data observed by weather stations located in a specific region of northwestern Spain (Galicia) to estimate the Weibull parameters using seven fitting methods. Among these seven methods, only two stood out; namely, the Part Density Energy Method (PDEM) and the Moments Method (MOM), which produced the best results (Carrilo, et al., 2014). Accurate estimation of wind speed distributions is a challenging task in wind power planning and operation. The selection of convenient functions for describing wind speed distribution is a crucial requisite (Wadi and Elmasry, 2021).

The main goals of this research have been to estimate both the parameters of the Weibull statistical model and the wind power density using wind data from three cities in northeastern Brazil. Additionally, the aim has been to analyze the best fit between the distribution of the observed data and the Weibull model.

In order to achieve all of these objectives, four different methods — Least-Squares Regression Method (LSRM), Moments Method (MOM), Energy Pattern Factor Method (EPFM), and Maximum Likelihood Method (MLM) — have been employed to estimate the shape and scale parameters of the Weibull statistical model. Additionally, wind speed data collected in three cities located in northeastern Brazil (Natal/RN, Maceió/AL, and Fortaleza/CE) have been used.

It should be noted that the values of the shape and scale parameters were estimated through four different methods, and only their mean values were utilized to estimate the Wind Power Density (WPD) in three cities in northeastern Brazil. Subsequently, the accuracy of the four methods was quantified using the Root Mean Square Error (RMSE) method. In other words, the RMSE statistical test was employed to analyze the best fit between the observed data and the Weibull statistical model.

2 Data and methods

The wind speed is a random variable that is difficult to forecast. However, wind speed data are of fundamental importance for estimating the wind potential in a locality or region. In this research, three time series of mean hourly wind data at 10 meters above the ground, collected by automatic stations administered by the National Meteorological Institute, were used. The three cities studied are located in the Brazilian Northeast, as shown in Table 1.

Table 1. Geographic location of the weather stations in the Brazilian northeast at 10 meters above ground level (2018).

City/State	Lat.(°)	Lon.(°)	Alt.(m)	Period
Natal/RN	-5,8372	-35,2079	47	Jan-Dec
Maceió/AL	-9,5512	-35,7702	84	Jan-Dec
Fortaleza/CE	-3,8157	-38,5378	30	Jan-Dec

To estimate the wind speed in the cities indicated in Table 1, various probability distribution models could be used, with the Weibull and Rayleigh distributions being notable among them. The shape parameter in the Weibull model indicates the type of probability distribution that best fits the wind data. Specifically, estimated values for the shape parameter less than two, equal to two, and greater than three are associated with the probability distribution of Weibull, the probability distribution of Rayleigh, and the probability distribution of Gaussian, respectively (e.g., Bidaoui. et al., 2019; Jiajin and Zhentong, 2023).

In particular, this work utilized the Weibull probability density function (PDFw), which has been the most widely used statistical model globally due to its best fit to observed wind speed data. In his 1951 paper, Swedish physicist Ernest Hjalmar Wallodi Weibull (1887-1979) sought to address the main objections related to the use of PDFw, given its lack of theoretical foundations. In this context, Weibull aimed to demonstrate that his probability distribution model could provide a best fit to observed data than many other already-known distributions (Weibull, 1951: Kumar and Gaddada, 2015; Kumar, et al., 2018)

In particular, the PDFw with two parameters (i.e., the shape and scale parameters) was employed to study the local wind regime. Thus, the PDFw, represented here by $f(v)$, has been characterized by its shape k and scale c parameters, as shown below (Pobocíková and Sedliacková, 2014; Kumar et al., 2018).

$$f(v) = \begin{cases} \dfrac{k}{c}\left[\dfrac{v}{c}\right]^{k-1} \exp\left[-\left(\dfrac{v}{c}\right)^{k}\right], & para\ v > 0 \quad e \quad k, c > 0; \\ 0 \quad , & para\ v \leq 0. \end{cases} \tag{1}$$

On the other hand, Weibull's cumulative probability density function, *F(v)*, represents the probability of occurrence for wind velocity events *V*, considering a value of *V* equal to or lower than *V*, as shown in Eq.2 below (e.g., Weibull, 1951).

$$F(v) = P(V \leq v) = \int_0^v f(v)dv = 1 - e^{-\left(\frac{v}{c}\right)^k} \tag{2}$$

The Weibull distribution has been extensively employed by the scientific community to analyze the probability of occurrence of random variables, such as wind speed. In this context, numerous authors, through their scientific publications, have utilized and compared various methods for estimating the parameters of the Weibull distribution. Their objective has been to identify a method that yields superior results in terms of accuracy.

Pobocíková and Sedliacková (2014) have described and also compared four different methods for estimating the parameters of the Weibull distribution, which were: the least square method (LSM), weighted least square method (WLSM), maximum likelihood method (MLM), and the method of moments (MOM). The results from the work of the above-mentioned authors have shown that the MOM provides estimates very similar to those obtained by the MLM. There is one complication in using the MOM. This method needs to use the gamma function. However, the gamma function can be easily obtained by using the software. The performance of the MLM is often better than the MOM. The MLM is the most popular for its efficiency, good properties, and it is simpler to compute than the MOM. Thus, the authors recommend using the MLM to estimate the Weibull distribution parameters.

Akdag and Dinler (2009) compared both the maximum likelihood method (MLM) and the graphical method (GM) with the energy pattern factor method (EPFM), where it was observed that the energy pattern factor method has presented the best estimates for both wind power and wind speed.

Indhumathy et al. (2014) have compared four methods for estimating the parameters of the Weibull probability distribution using wind data from India. Thus, the results have shown that the accuracy of these four methods may change according to the sample data, i.e., with the data sample size, the distribution of the sample data, and the format of the sample data. Another result has shown that the Energy Pattern Factor Method (EPFM) was the most efficient method to determine the shape (k) and scale (c) parameters because it presented the best fit between the Weibull model and the observed wind speed data in India.

In particular, in this work, four different methods to estimate the parameters of the Weibull distribution using hourly wind velocity data in three cities in Northeast Brazil (i.e., Maceió-AL, Fortaleza-CE, and Natal-RN) were analyzed and compared. The four methods employed were the Energy Pattern Factor Method (EPFM), Method of Moments (MOM), Least-Squares Regression Method (LSRM), and Maximum Likelihood Method (MLM).

2.1 Energy pattern factor method (EPFM)

The Energy pattern factor method EPFM is characterized by its relatively easier understanding and implementation. Initially, the energy pattern factor *Epf*, defined as the ratio between the mean cubic wind speed and the cube of the mean wind speed, was calculated as follows (Kumar and Gaddada, 2015; Shu and Jesson, 2021).

$$Epf = \frac{\overline{v^3}}{(\bar{v})^3} = \frac{\Gamma(1+3/k)}{\Gamma^3(1+1/k)}, \tag{3}$$

where k is the shape parameter and Γ is the gamma function defined as

$$\Gamma(x) = \int_0^\infty t^{x-1} \exp(-t)dt. \tag{4}$$

Initially, the *Epf* was calculated using Eq.3, and then the shape k and scale c parameters were calculated through two equations, as follows.

$$k = 1 + \frac{3{,}69}{E_{pf}^2} \tag{5}$$

$$c = \frac{\bar{v}}{\Gamma(1+1/k)} \tag{6}$$

2.2 Moments method (MOM)

In order to apply the Moment Method (MOM), both the mean velocity and the standard deviation must initially be calculated using the observed data. Thus, for the estimation of the shape k and scale c parameters from the Weibull distribution, the following equations were employed (Chaurasiya et al., 2018; Kumar et al., 2019).

$$\bar{v} = c\Gamma\left(1+\frac{1}{k}\right) \tag{7}$$

$$\frac{\sigma}{\bar{v}} = \sqrt{\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1} \tag{8}$$

where σ is standard deviation and $\bar{v}$ mean speed from observed data.

However, it's necessary to highlight that Eq.8 cannot be solved analytically with respect to k. It requires a numerical method for solving it. After calculating the k parameter, it's then easily possible to determine the c parameter using Eq.7.

2.3 Least-squares regression method (LSRM)

In cases where the MOM shows unsatisfactory results for estimating the parameters of a statistical distribution model, another estimation method can be employed. In this context, the Least-Squares Regression Method (LSRM) may be a great alternative for estimating the parameters of the Weibull model. LSRM is also known as the graphical method, where the cumulative Weibull distribution must be linearized to find a straight line that best fits the observed wind data distribution (Kumar et al., 2015; Chaurasiya et al., 2017).

Let {v1, v2,...,vn} be a random sample of size n from the Weibull distribution. Thus, according to the LSRM, the cumulative distribution function F(v) must be transformed into a linear function by applying the double logarithm to both sides of Eq.2, as shown in Eq.9 below,

$$\ln\{-\ln[1-F(v)]\} = k\ln(v) - k\ln c \tag{9}$$

Looking at Eq.9, it's possible to observe its linearity through a change of variables. That is, if we consider Y = ln{-ln[1-F(v)]}, X = ln(v), a = k, and b = -kln(c), then Eq.9 may be rewritten as follows:

$$Y = aX + b \tag{10}$$

To estimate a set of values using the cumulative distribution function F(v), the Herd-Johnson's estimator, also known as the mean rank estimator, was employed, as shown below (Chaurasiya et al., 2018):

$$\hat{\mathrm{F}}(\mathrm{v}) = \frac{\mathrm{i}}{N+1}, \tag{11}$$

where i denotes the ith rank of each v_i random variable, i = 1, 2, ..., n and N is the total number of observations. Particularly, linear regression coefficients a and b calculated by their estimators $\hat{a}$ and $\hat{b}$ must minimize the following function

$$\mathrm{T(k,c)} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \left(\mathrm{y_i} - a\ln(v_i) - b\right)^2, \tag{12}$$

so, the estimators $\hat{a}$ and $\hat{b}$ have been useful in estimating the linear regression coefficients a and b, as shown below:

$$\hat{a} = \frac{N\sum_{i=1}^{N}\ln(v_i)\ln\left[-\ln\left(1-\hat{F}(v_i)\right)\right]-\sum_{i=1}^{N}\ln(v_i)\sum_{i=1}^{N}\ln\left[-\ln\left(1-\hat{F}(v_i)\right)\right]}{N\sum_{i=1}^{N}\ln^2(v_i)-\left(\sum_{i=1}^{N}\ln(v_i)\right)^2}, \tag{13}$$

$$\hat{b} = \frac{1}{N}\sum_{i=1}^{N}\ln\left[-\ln\left(1-\hat{F}(v_i)\right)\right]-\hat{c}\frac{1}{N}\sum_{i=1}^{N}\ln(v_i). \tag{14}$$

Finally, the estimates of the shape k and scale c parameters from the Weibull model are shown as follows (Chaurasiya et al., 2018; Pobocíková and Sedliacková, 2014):

$$k = \hat{a}, \tag{15}$$

$$c = e^{-\left(\frac{\hat{b}}{k}\right)}. \tag{16}$$

2.4 Maximum likelihood method (MLM)

The Maximum Likelihood Method (MLM) has been widely employed for estimating the parameters of the Weibull probability distribution. However, MLM is a more complex method, so it requires the use of some iterative methods to estimate the Weibull parameters. In general, numerous studies have demonstrated, through various statistical fit tests, that MLM exhibits good accuracy in estimating the parameters of the Weibull distribution. Specifically, the likelihood function applied to the Weibull probability density function (PDFw) is shown as follows (Azad,et al., 2014; Chaurasiya et al., 2018):

$$L(k,c) = \prod_{i=1}^{N} f(v_i,|k,c|) = \prod_{i=1}^{N}\frac{k}{c}\left[\frac{v_i}{c}\right]^{k-1}\exp\left[-\left(\frac{v_i}{c}\right)^k\right] \tag{17}$$

When the logarithm is applied to both sides of Eq.17, it's possible to obtain Eq.18, as follows.

$$lnL(c,k) = Nln(k) - Nkln(c) + (k-1)\sum_{i=1}^{N}\ln(V_i) - \frac{1}{c^k}\sum_{i=1}^{N}V_i^k \tag{18}$$

The shape k and scale c parameters of the Weibull distribution can be estimated by Eq.18, which must be initially differentiated with respect to c and then, with respect to k, to obtain Eq.19 and Eq.20 respectively, as follows:

$$-\frac{Nk}{c}+\frac{k}{c^{k+1}}\sum_{i=1}^{N}v_i^k = 0, \tag{19}$$

$$\frac{N}{k} - N\ln c - \frac{\sum_{i=1}^{N}v_i^k\ln v_i - \ln c\sum_{i=1}^{N}v_i^k}{c^k}+\sum_{i=1}^{N}\ln v_i = 0. \tag{20}$$

After rearranging Eq.19 and eliminating c in Eq.20, it's possible to obtain the following equations:

$$c = \left(\frac{1}{N} \sum_{i=1}^{N} v_i^k \right)^{1/k}, \tag{21}$$

$$\frac{1}{k} - \frac{\sum_{i=1}^{N} v_i^k \ln v_i}{\sum_{i=1}^{N} v_i^k} + \frac{1}{N} \sum_{i=1}^{N} \ln v_i = 0. \tag{22}$$

The Eq.22 cannot be solved analytically, so its solution requires an iterative method for estimating the shape parameter *k*. On the other hand, with the knowledge of the value of k, Eq.21 can be easily solved.

2.5 Statistical analysis

According to the literature, various methods have been employed to determine the best fit between the cumulative distribution of wind data Fo(vi) and the cumulative probability distribution of the Weibull model Fe(vi). In other words, the precision of estimating the shape (k) and scale © parameters of the Weibull probability distribution can be analysed through diverse techniques, including the Chi-square test, Kolmogorov–Smirnov test, relative percentage of error, mean percentage error, absolute mean percentage error, or root-mean-square error (Azad, et al., 2014; Chaurasiya, et al., 2018, Kumar, et al., 2019). However, in this study, the Root-mean-square error method (RMSE) was selected and is presented below.

$$RMSE = \left[\frac{1}{N} \sum_{i=1}^{N} \left(F_o(v_i) - \hat{F}_e(v_i) \right)^2 \right]^{\frac{1}{2}}. \tag{23}$$

2.6 Wind power density

The estimation of local wind power density is fundamental before deciding to implement any wind energy conversion systems, such as wind farms. In this paper, the Weibull distribution has been used to estimate both the mean $\bar{v}$ and the variance σ^2 of the wind speed. The method of moments has been used to derive Eq. 24 and Eq. 25 as follows (Kumar et al., 2019; Rocha et al., 2012).

$$\bar{v} = c\Gamma\left(1 + \frac{1}{k}\right) \tag{24}$$

$$\sigma^2 = c^2\Gamma\left(1 + \frac{2}{k}\right) - \left[c\Gamma\left(1 + \frac{1}{k}\right)\right]^2 \tag{25}$$

In other words, a study on wind characteristics must be conducted before the implementation of any wind farms to optimize both the occupied area and the capacity factor. Specifically, the wind turbine is a conversion system that transforms the kinetic energy from the wind flow into electrical energy. In this context, the Wind Power Density (WPD) has been estimated, as follows (e,g., Manwell, et al., 2009; Pishgar-Komleh et al., 2015):

$$WPD = \frac{1}{2}\rho c^3\Gamma\left(1 + \frac{3}{k}\right), \tag{26}$$

where the adopted mean value for the atmospheric air density in the studied region was ρ = 1.21 kg/m³.

3 Results and discussions

3.1 Weibull parameters analysis

The estimates for the parameters of Weibull's distribution, obtained using all four methods mentioned in the previous section, have been crucial for estimating the wind power density (WPD) in the Brazilian

Northeast. In this section, accuracies in the calculation of both the shape k and scale c parameters of Weibull's distribution were analyzed using time series data collected in three cities from the northeastern region of Brazil, namely Maceió-AL, Fortaleza-CE, and Natal-RN. Analyzing Figures 2, 3, and 4, it is evident that the shape k and scale c parameters of Weibull's distribution show temporal variabilities. Specifically, the scale parameter c has been estimated using four different methods, and its calculated values demonstrated greater convergence when compared to the shape parameters k. In other words, during the twelve months analyzed, the methods (LSRM, MLM, MOM, and EPFM) showed the best precision for estimating the scale parameter c compared to the shape parameter k. Particularly, Figure 4 shows, based on the larger relative dispersion of points in the graph, that the shape parameter k, estimated by the four mentioned methods, presented lower precision in November in the city of Natal.

Analyzing the temporal variability of the shape parameter k in Fortaleza city, the highest precision was observed in August, while the lowest precision occurred in July, as depicted in Figure 2. Conversely, upon examining Figures 3 and 4, it is evident that the greatest precisions for the k shape parameter were observed in the cities of Maceió, AL, and Natal, RN, in February and April, respectively.

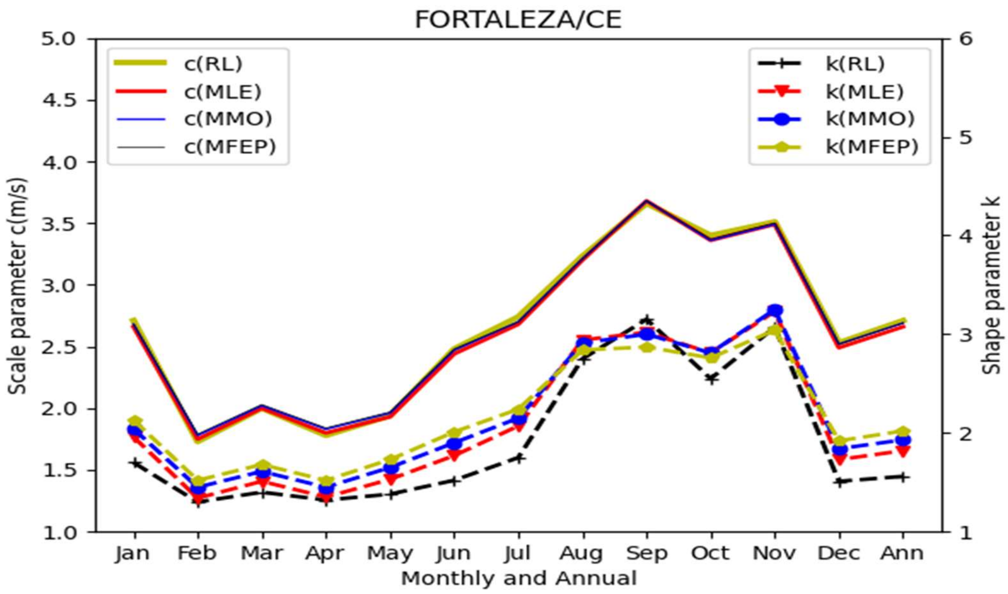


Figure 2. Monthly variability of the estimated shape (k) and scale (c) parameters in Fortaleza city.

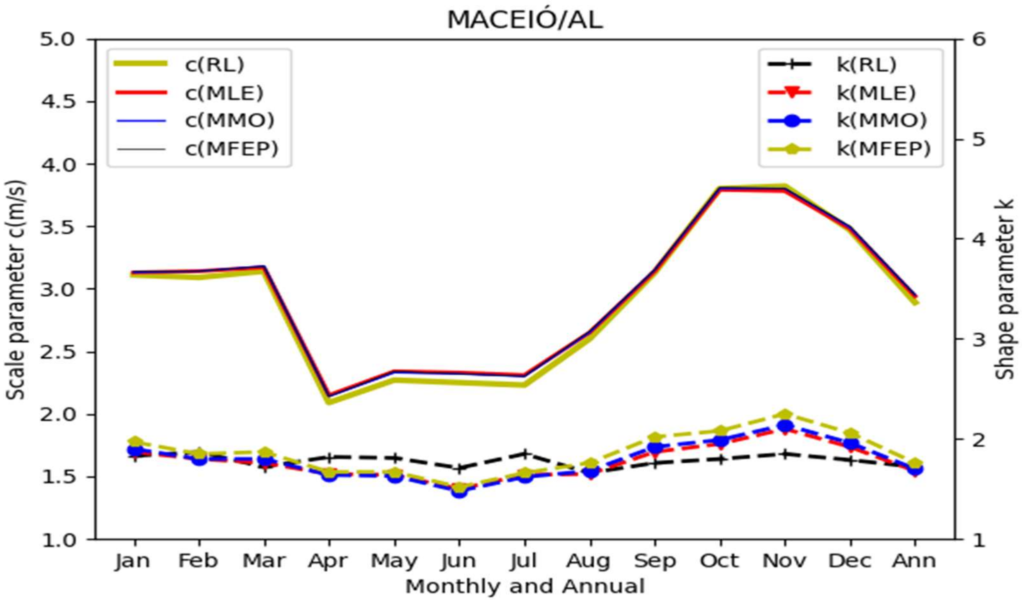


Figure 3. Monthly variability of the estimated shape (k) and scale (c) parameters in Maceió city.

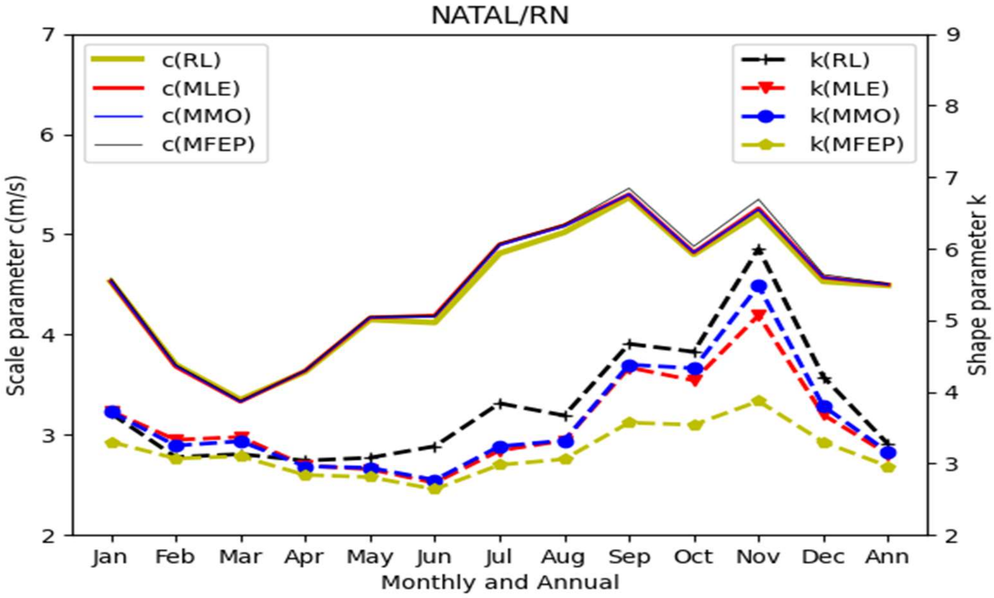


Figure 4. Monthly variability of the estimated shape (k) and scale (c) parameters in Natal city.

To assess the best fit between the observed data's cumulative distribution, Fo(vi), and the Weibull cumulative probability distribution, Fe(vi), the Root Mean Square Error (RMSE) test was employed.

Therefore, the best fits have been ranked from first to fourth positions, in this order, as shown in Table 2. In Table 2, it is observed that the RMSE (Fortaleza/CE) based on the MLM reached the first position in the ranking (i.e., best fit) in four out of the twelve months of the analyzed year. Put differently, in the city of Fortaleza, CE, the MLM secured the first position for the best fit in 33.33% of the assessed months, followed by LSRM (25.00%), EPFM (25.00%), and MOM (16.66%) methods. However, for the city of Maceió, AL, both LSRM and MOM methods tied for the first position, presenting the best fits in 41.66% of the assessed months, while the MLM method took the third position with 16.7%.

Finally, the EPFM method 129the reached the third or fourth positions in the ranking of the best fits in all months. Specifically, in the city of Natal, RN, the EPFM method reached the first position in the ranking for the best fit in 41.66% of the analyzed months, followed by MLM (33.33%), MOM (16.66%), and LSRM (0.08%) methods. In general, subsequent to the analysis of the results presented in Table 2, it was possible to ascertain that the accuracies in estimating Weibull's parameters, employing four distinct methods, have exhibited spatial and temporal variability.

Table 2. Ranking of the best fit among the four methods (LSRM, MLM, MOM, and EPFM) using the Root Mean Square Error (RMSE) test. The best fits are ranked from 1st to 4th positions.

	RMSE (Fortaleza/CE)				RMSE (Maceió/AL)				RMSE (Natal/RN)			
	LSRM	MLM	MOM	EPFM	LSRM	MLM	MOM	EPFM	LSRM	MLM	MOM	EPFM
Jan	0,2394	0,1718	0,1775	0,1990	0,2700	0,2795	0,2880	0,3207	0,0959	0,0955	0,0944	0,1668
	(4th)	(1st)	(2nd)	(3rd)	(1st)	(2nd)	(3rd)	(4th)	(3rd)	(2nd)	(1st)	(4th)
Feb	0,2028	0,2059	0,2429	0,2781	0,3197	0,2714	0,2726	0,3010	0,1314	0,1161	0,1125	0,1317
	(1st)	(2nd)	(3rd)	(4th)	(4th)	(1st)	(2nd)	(3rd)	(3rd)	(2nd)	(1st)	(4th)
Mar	0,1965	0,2004	0,2267	0,2566	0,3235	0,3284	0,3413	0,3747	0,1395	0,1014	0,1051	0,1405
	(1st)	(2nd)	(3rd)	(4th)	(1st)	(2nd)	(3rd)	(4th)	(3rd)	(1st)	(2nd)	(4th)
Apr	0,2182	0,2200	0,2552	0,2904	0,4055	0,3205	0,3116	0,3293	0,1185	0,1116	0,1081	0,1014
	(1st)	(2nd)	(3rd)	(4th)	(4th)	(2nd)	(1st)	(3rd)	(4th)	(3rd)	(2nd)	(1st)
May	0,2368	0,2189	0,2352	0,2620	0,4290	0,3348	0,3285	0,3508	0,1834	0,1656	0,1666	0,1647
	(3rd)	(1st)	(2nd)	(4th)	(4th)	(2nd)	(1st)	(3rd)	(4th)	(2nd)	(3rd)	(1st)
Jun	0,3439	0,2950	0,2933	0,3153	0,4768	0,3440	0,3308	0,3548	0,3656	0,2946	0,2941	0,2896
	(4th)	(2nd)	(1st)	(3rd)	(4th)	(2nd)	(1st)	(3rd)	(4th)	(3rd)	(2nd)	(1st)
Jul	0,3063	0,2050	0,1988	0,2086	0,4661	0,3446	0,3336	0,3549	0,4356	0,3101	0,3135	0,2947
	(4th)	(2nd)	(1st)	(3rd)	(4th)	(2nd)	(1st)	(3rd)	(4th)	(2nd)	(3rd)	(1st)
Aug	0,0965	0,0915	0,0876	0,0794	0,3413	0,3231	0,3384	0,3734	0,2933	0,2111	0,2054	0,1820
	(4th)	(3rd)	(2nd)	(1st)	(3rd)	(1st)	(2nd)	(4th)	(4th)	(3rd)	(2nd)	(1st)
Sep	0,1933	0,1680	0,1580	0,1383	0,3227	0,3385	0,3478	0,3804	0,1718	0,1363	0,1384	0,2722
	(4th)	(3rd)	(2nd)	(1st)	(1st)	(2nd)	(3rd)	(4th)	(3rd)	(1st)	(2nd)	(4th)
Oct	0,1395	0,0853	0,0880	0,0845	0,2813	0,2941	0,3008	0,3301	0,1568	0,1720	0,1657	0,3271
	(4th)	(2nd)	(3rd)	(1st)	(1st)	(2nd)	(3rd)	(4th)	(1st)	(3rd)	(2nd)	(4th)
Nov	0,1866	0,1740	0,1790	0,1791	0,3307	0,3242	0,3240	0,3475	0,2400	0,2271	0,2275	0,4773
	(4th)	(1st)	(2nd)	(3rd)	(3rd)	(2nd)	(1st)	(4th)	(3rd)	(1st)	(2nd)	(4th)
Dec	0,2556	0,2007	0,2074	0,2316	0,3234	0,3345	0,3411	0,3700	0,2493	0,2207	0,2229	0,2742
	(4th)	(1st)	(2nd)	(3rd)	(1st)	(2nd)	(3rd)	(4th)	(3rd)	(1st)	(2nd)	(4th)
Ann.	0,3212	0,2209	0,2120	0,2272	0,3249	0,2961	0,3072	0,3367	0,1336	0,1242	0,1256	0,1422
	(4th)	(2nd)	(1st)	(3rd)	(3rd)	(1st)	(2nd)	(4th)	(3rd)	(1st)	(2nd)	(4th)

In other words, Table 2 summarizes the ranking of the best estimators for Weibull distribution parameters. The MLE was the best estimator in 33.3% of the months analyzed in Fortaleza/CE. The EPFM emerged as the best estimator in 41.7% of the months examined in Natal/RN. Finally, the LSRM and MOM are tied, as 130the were the best estimators in 41.7% of the months analyzed in Maceió. For the annual period, MLM was the best estimator in two out of three cities examined.

Table 3 presents the temporal distributions of the mean values for the shape $\bar{k}$ and scale $\bar{c}$ parameters estimated by the four methods analyzed in the previous section. Additionally, it includes the wind mean speed $\bar{V}$ estimate based on the mean values of these parameters.

Table 3. Mean values for both wind speed $\bar{V}$ and the Weibull parameters ($\bar{k}$, $\bar{c}$) estimated using the following methods: EPFM, MOM, LSRM, MLM.

	Fortaleza-CE			Maceió-AL			Natal-RN		
	$\bar{c}$(m/s)	$\bar{k}$	$\bar{V}$	$\bar{c}$(m/s)	$\bar{k}$	$\bar{V}$	$\bar{c}$(m/s)	$\bar{k}$	$\bar{V}$
Jan	2,6826	1,9279	2,3794	3,1259	1,8906	2,7742	4,5379	3,5784	4,0878
Feb	1,7606	1,3876	1,6069	3,1245	1,8392	2,7758	3,6941	3,1627	3,3068
Mar	2,0062	1,5302	1,8069	3,1674	1,7845	2,8179	3,3377	3,1989	2,9894
Apr	1,8074	1,3962	1,6480	2,1246	1,7166	1,8945	3,6378	2,9533	3,2462
May	1,9447	1,5485	1,7491	2,3197	1,7056	2,0693	4,1660	2,9354	3,7167
Jun	2,4667	1,7708	2,1955	2,3001	1,5797	2,0647	4,1649	2,8694	3,7122
Jul	2,7089	2,0186	2,4003	2,2794	1,7160	2,0325	4,8731	3,3344	4,3735
Aug	3,2129	2,8437	2,8626	2,6373	1,6874	2,3544	5,0694	3,3501	4,5508
Sep	**3,6745**	3,0161	**3,2820**	3,1378	1,8842	2,7852	**5,4079**	4,1924	**4,9149**
Oct	3,3740	2,7076	3,0007	3,7991	1,9408	**3,3691**	4,8349	4,0838	4,3876
Nov	3,5023	**3,1143**	3,1328	**3,8016**	**2,0693**	3,3675	5,2751	**4,9800**	4,8423
Dec	2,5153	1,7216	2,2424	3,4840	1,9211	3,0905	4,5682	3,7333	4,1246
Ann. Mean	2,6865	1,7996	2,3891	2,9304	1,7198	2,6126	4,5009	3,1184	4,0262

The highest values for both Weibull's parameters and mean speed were observed between September and November in the three analyzed cities. On the other hand, upon analysing Figures 3 and 4, it's easy to see that the greatest precisions occurred in the cities of Maceió/AL and Natal/RN in February and April, respectively. However, the highest values for the shape parameter k occurred in November. The peak values for the estimated mean wind speeds were observed in both September (Fortaleza/CE and Natal/RN) and October (Maceió/AL). In general, by analyzing Table 3, it was observed that the highest estimated values for both mean speeds and Weibull parameters occurred between September and November in the three cities located in the Brazilian northeast.

3.2 Wind power density WPD and variance VaR

Important information regarding both the temporal variability of the wind power density (WPD) and the variance (VaR) in three cities of the Brazilian Northeast is presented in Figure 5. It is evident, through comparisons among the three cities, that the highest values for the variance of wind speed (VaR_M) occurred in the city of Maceió/AL in 58.3% of the analyzed months. On the other hand, in the city of Natal/RN, the highest estimated values for the variances of wind speed (VaR_N) occurred in 41.7% of the months. It's interesting to highlight that in Fortaleza/CE, the estimated value for the variance of wind speed (VaR_F) was not the highest in any of the months, in comparison with the other two cities. Particularly, upon conducting comparisons between the municipalities of Fortaleza/CE and Natal/RN, it was observed that the estimated variance of wind speed in Fortaleza/CE surpassed that in Natal/RN in approximately 41.7% of the months under analysis.

Using the annual mean value as a reference for the variance of wind speed in the three analyzed cities, it was observed that the estimate for the variance of wind speed in Maceió/AL was the highest. This is due to an increment above the mean, approximately 18%. Contrastingly, concerning the other two cities, it was observed that the estimates for the variance of wind speed were below the mean by approximately 13.0% (Fortaleza/CE) and 5.0% (Natal/RN). Generally, variance is a pivotal variable as it can serve as an indicator of wind quality. In other words, the estimated temporal variability for the variance of the data has shown that the wind has exhibited the highest quality in Fortaleza/CE city, followed by Natal/RN and Maceió/AL, in that order.

By analyzing the monthly variability for wind power density (WPD), it was verified that its maximum estimated values occurred in September (Natal/RN and Fortaleza/CE) and October (Maceió/AL), and these results are in accordance with the Annual Wind Energy Report 2022.

Comparing the estimated maximum values of Wind Power Density (WPD), it was determined that WPD(N) in Natal surpassed both WPD(F) in Fortaleza and WPD(M) in Maceió/AL by approximately 290% and 192%, respectively. Over the annual period, it was observed that the estimate for WPD(N) exceeded both WPD(F) and WPD(M) by about 315% and 221%, respectively.

Through an analysis of the results summarized in Figure 5, it is evident that the Wind Power Density (WPD(N)) in Natal/RN has surpassed that of Fortaleza/CE and Maceió/AL in all months, except for March, which was more favorable for the city of Maceió. In other words, the presented results have indicated that in March alone, the WPD(M) in Maceió exceeded both WPD(N) and WPD(F) by approximately 34% and 218%, respectively.

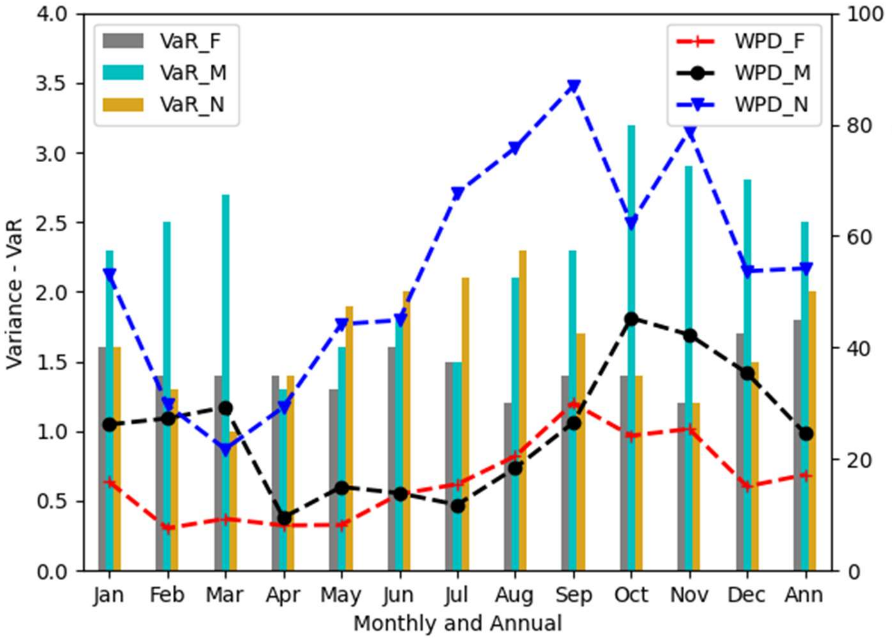


Figure 5. Temporal variations in Variance (VaR) and Wind Power Density (WPD) for the cities of Fortaleza (F), Maceió (M), and Natal (N).

4 Conclusion

The electricity generation from renewable sources has been a worldwide trend, where, in this context, wind power has shown its importance in order to diversify and strengthen the world electrical matrix, which is currently dominated by fossil fuels (coal, oil, and gas) that produce carbon dioxide and other greenhouse gases.

This work, in particular, was dedicated to studying the wind power estimate, where it was evidenced that the winds that blow in the Brazilian northeast have been favorable to wind energy production. Specifically, all the results presented in this paper have shown that:

1. Considering the twelve months analyzed, the LSRM, MLM, MOM, and EPFM methods have shown more accurate results for estimating the scale parameter c compared to the shape parameter *k*;
2. Analyzing the fit between the observed data distribution and the Weibull model, it was found that for the cities of Fortaleza/CE and Natal/RN, the best fits were achieved by the MLM and EPFM methods, respectively. However, in the city of Maceió/AL, there was a tie between the LSRM and MOM methods regarding the best fit in 41.7% of the months;
3. The highest estimated values for both the Weibull parameters and the mean speed were observed between the months of September and November, considering all three analyzed cities;

4. The city of Fortaleza/CE has presented the best wind quality, followed in descending order by both Natal/RN and Maceió/AL cities. Part of the achieved results has shown that both the Weibull parameters and the estimated Wind Power Density WPD have spatial and temporal variability;
5. In analyzing the monthly variability of Wind Power Density (WPD), it was observed that its maximum estimated values occurred in September (Natal/RN and Fortaleza/CE) and October (Maceió). These results correspond with the reported peaks in wind power generation as shown in the Annual Wind Energy Report of 2018;
6. The city of Natal/RN has exhibited the highest values for the estimated Wind Power Density (WPD) in comparison to Fortaleza/CE and Maceió/AL cities;
7. The winds blowing in the Brazilian northeast have demonstrated excellent quality, with the most favorable conditions for wind power generation occurring more frequently in the second half of the year;
8. The results obtained in this study may inspire many other researchers to enhance the estimation of wind power density in different regions of Brazil and around the world.

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